

Symmetry energy systematics and its high density behavior

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Abstract. We explore the systematics of the density dependence of nuclear matter symmetry energy in the ambit of microscopic calculations with various energy density functionals, and find that the symmetry energy from subsaturation density to supra-saturation density can be well determined by three characteristic parameters of the symmetry energy at saturation density ρ_0 , i.e., the magnitude $E_{\text{sym}}(\rho_0)$, the density slope L and the density curvature K_{sym} . This finding opens a new window to constrain the supra-saturation density behavior of the symmetry energy from its (sub-)saturation density behavior. In particular, we obtain $L = 46.7 \pm 12.8$ MeV and $K_{\text{sym}} = -166.9 \pm 168.3$ MeV as well as $E_{\text{sym}}(2\rho_0) \approx 40.2 \pm 12.8$ MeV and $L(2\rho_0) \approx 8.9 \pm 108.7$ MeV based on the present knowledge of $E_{\text{sym}}(\rho_0) = 32.5 \pm 0.5$ MeV, $E_{\text{sym}}(\rho_c) = 26.65 \pm 0.2$ MeV and $L(\rho_c) = 46.0 \pm 4.5$ MeV at $\rho_c = 0.11 \text{ fm}^{-3}$ extracted from nuclear mass and the neutron skin thickness of Sn isotopes. Our results indicate that the symmetry energy cannot be stiffer than a linear density dependence. In addition, we also discuss the quark matter symmetry energy since the deconfined quarks could be the right degree of freedom in dense matter at high baryon densities.

1 Introduction

The nuclear matter symmetry energy, which essentially characterizes the isospin dependent part of the equation of state (EOS) of asymmetric nuclear matter, is important for understanding many questions in nuclear physics and astrophysics, including the nuclear effective interactions in asymmetric nuclear matter, the structure and stability of exotic nuclei, the reaction dynamics induced by rare isotopes, the nature and evolution of neutron stars, and the mechanism of supernova explosion [1–7]. The symmetry energy also plays an important role in some interesting issues of new physics beyond the standard model [8–12]. During the last decade, a lot of experimental, observational and theoretical efforts have been devoted to constraining the density dependence of the symmetry energy [13–18]. While significant progress has been made in determining the density behavior of the symmetry energy around saturation density ρ_0 ($\sim 0.16 \text{ fm}^{-3}$), its supra-saturation density behavior is still poorly known and remains the most uncertain property of isospin asymmetric nuclear matter. Theoretically, many experimental and observational probes have been proposed to extract information on the supra-saturation density behavior of the symmetry energy [13]. In terrestrial laboratories,

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heavy-ion collisions provide the only way to explore the supra-saturation density behavior of the symmetry energy under controlled conditions. To the best of our knowledge, the constraints on the supra-saturation density behavior of the symmetry energy obtained so far are all from the transport model analyses on the data of π^-/π^+ ratio [19–25] and n/p elliptic flows [26, 27] in heavy-ion collisions but unfortunately they are contradictory with each other, leaving a confusing situation for the community.

Conventionally, the nuclear matter EOS is defined as the binding energy per nucleon as a function of the density and a number of bulk characteristic parameters defined at saturation density ρ_0 are usually introduced to quantitatively characterize the energy of symmetric nuclear matter and the symmetry energy [28, 29]. For example, the energy $E_0(\rho_0)$ and incompressibility K_0 of symmetric nuclear matter are the two lowest order bulk parameters for the EOS of symmetric nuclear matter while the symmetry energy magnitude $E_{\text{sym}}(\rho_0)$ and its slope parameter L are the two lowest order bulk parameters of the symmetry energy. While several lower order bulk characteristic parameters of asymmetric nuclear matter, such as $E_0(\rho_0)$, K_0 , $E_{\text{sym}}(\rho_0)$ and L have been relatively well constrained or in significant progress [13–18, 30], yet the higher order bulk characteristic parameters are still poorly known. Actually, there has little experimental information on the third-order derivative parameter J_0 of symmetric nuclear matter at ρ_0 [31] and the symmetry energy curvature parameter K_{sym} [29]. However, the higher order bulk characteristic parameters have been shown to be closely related to some important issues in nuclear physics and astrophysics, such as the determination of the isobaric incompressibility of asymmetric nuclear matter [28, 32] and the core-crust transition density and pressure in neutron stars [33–35]. In particular, within the Skyrme energy density functional, it has been proposed [29] that the higher-order curvature parameter K_{sym} may play an important role in the determination of the supra-saturation density behaviors of the symmetry energy.

So far (very likely also in future), essentially all the obtained constraints on $E_{\text{sym}}(\rho)$ are based on some energy density functionals or phenomenological parameterizations of $E_{\text{sym}}(\rho)$. Therefore, it would be very interesting to see whether there exist some universal laws (systematics) for the density dependence of the symmetry energy within these functionals or parameterizations and whether one can get some useful information on the high density symmetry energy from the relatively well-known knowledge of the symmetry energy around saturation density. For all the energy density functionals or phenomenological parameterizations, the $E_{\text{sym}}(\rho)$ increases from $\rho = 0$ up to a certain density around ρ_0 and then either continuously increases or decreases depending on the parameters of the energy density functionals or phenomenological parameterizations. While the parameters $E_{\text{sym}}(\rho_0)$, L and K_{sym} accurately characterize the symmetry energy density behaviors around ρ_0 , their relation to the density behaviors at sub- and supra-saturation densities in various energy density functionals or phenomenological parameterizations of $E_{\text{sym}}(\rho)$ are still unclear.

In the present talk, we report the preliminary results of the study on the systematics of the density dependence of nuclear matter symmetry energy in the ambit of microscopic calculations with various energy density functionals. We systematically analyze the relation between the parameters $E_{\text{sym}}(\rho_0)$, L and K_{sym} defined at saturation density ρ_0 and the symmetry energy density behaviors at sub- and supra-saturation densities in various energy density functionals. In addition, since the dense matter at high baryon densities could be quark matter, we also discuss briefly the quark matter symmetry energy.

2 Symmetry energy systematics

The EOS of isospin asymmetric nuclear matter, given by its binding energy per nucleon, can be expanded to 2nd-order in isospin asymmetry δ as

$$E(\rho, \delta) = E_0(\rho) + E_{\text{sym}}(\rho)\delta^2 + O(\delta^4), \quad (1)$$

where $\rho = \rho_n + \rho_p$ is the baryon density with ρ_n and ρ_p denoting the neutron and proton densities, respectively; $\delta = (\rho_n - \rho_p)/\rho$ is the isospin asymmetry; $E_0(\rho) = E(\rho, \delta = 0)$ is the binding energy per nucleon in symmetric nuclear matter, and the nuclear symmetry energy is expressed as

$$E_{\text{sym}}(\rho) = \frac{1}{2!} \frac{\partial^2 E(\rho, \delta)}{\partial \delta^2} \Big|_{\delta=0}. \quad (2)$$

Around a reference density ρ_r , the $E_{\text{sym}}(\rho)$ can be expanded in $\chi_r = (\rho - \rho_r)/\rho_r$ as

$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_r) + \frac{L(\rho_r)}{3} \chi_r + \frac{K_{\text{sym}}(\rho_r)}{2!} \chi_r^2 + O(\chi_r^3), \quad (3)$$

where $L(\rho_r) = 3\rho_r \frac{\partial E_{\text{sym}}(\rho)}{\partial \rho} \Big|_{\rho=\rho_r}$ and $K_{\text{sym}}(\rho_r) = 9\rho_r^2 \frac{d^2 E_{\text{sym}}(\rho)}{d\rho^2} \Big|_{\rho=\rho_r}$ are, respectively, the slope and curvature parameters of the symmetry energy at ρ_r , and they are the lowest-order two bulk parameters characterizing the density behaviors of the symmetry energy around ρ_r . In particular, when the reference density ρ_r is taken as the saturation density ρ_0 , the $L(\rho_r)$ and $K_{\text{sym}}(\rho_r)$ are then reduced to the famous symmetry energy slope parameter $L \equiv 3\rho_0 \frac{dE_{\text{sym}}(\rho)}{d\rho} \Big|_{\rho=\rho_0}$ and symmetry energy curvature parameter $K_{\text{sym}} \equiv 9\rho_0^2 \frac{d^2 E_{\text{sym}}(\rho)}{d\rho^2} \Big|_{\rho=\rho_0}$, respectively.

To examine the symmetry energy systematics, we select a comprehensive large sample of 60 well-calibrated interactions in various energy density functionals, namely, 33 Skyrme interactions (v090, MSk7, BSk8, SKP, SKT6, SKX, BSk17, SGII, SKM*, SLy4, SLy5, MSkA, MSL0, SIV, SkSM*, kMP, SKa, Rsigma, Gsigma, SKT4, SV, SkI2, SkI5, BSk18, BSk19, BSk20, BSk21, MSL1, SAMi, SV-min, UNEDF0, UNEDF1, TOV-min), 2 Gogny interactions (D1S and D1N), 18 nonlinear RMF interactions (FSUGold, PK1s24, NL3s25, G2, TM1, NL-SV2, NL-SH, NL-RA1, PK1, NL3, NL3*, G1, NL2, NL1, IU-FSU, BSP, IUFSU*, TM1*), 2 density-dependent RMF interactions (DD-ME1 and DD-ME2), 3 point-coupling RMF interactions (DD-PC1, PC-PK1, PC-F1), and 2 relativistic HF interactions (PKO3 and PKA1). These interactions include the 46 interactions used in Ref. [36] (except BCP which is designed for density up to only 0.24 fm^{-3}) and other 14 interactions (i.e., BSk18, BSk19, BSk20, BSk21, MSL1, SAMi, SV-min, UNEDF0, UNEDF1, TOV-min, IU-FSU, BSP, IUFSU*, TM1*) constructed more recently. Shown in Fig. 1 is the symmetry energy as a function of the density normalized by the corresponding saturation ρ_0 with the 60 interactions. It is clearly seen that various energy density functionals predict very different density behaviors of the symmetry energy, especially at supra-saturation densities. For example, the magnitude of the symmetry energy at $2\rho_0$ can be varied from about 15 MeV to 100 MeV, depending on the models and interaction parameters. Furthermore, it is seen that some non-relativistic interactions predict negative symmetry energy at baryon densities above about $2.5\rho_0$.

Around the saturation density ρ_0 , the magnitude $E_{\text{sym}}(\rho)$ and the density slope $L(\rho)$ of the symmetry energy can be well approximated, respectively, by

$$E_{\text{sym}}^*(\rho) \equiv E_{\text{sym}}(\rho_0) + L\chi + \frac{K_{\text{sym}}}{2!} \chi^2, \quad (4)$$

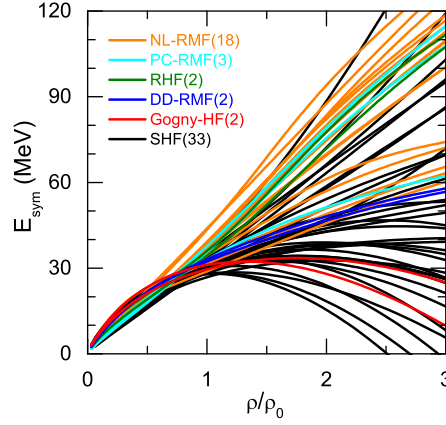


Figure 1. (Color online) The symmetry energy as a function of the density normalized by the corresponding saturation ρ_0 in various energy density functionals with 60 interactions. See text for the details.

and

$$L^*(\rho) \equiv L\rho/\rho_0 + K_{\text{sym}}\chi\rho/\rho_0, \quad (5)$$

with $\chi = (\rho - \rho_0)/\rho_0$. Based on a correlation analysis within SHF approach, it has been shown [29] that $E_{\text{sym}}^*(\rho)$ can well describe the magnitude of the symmetry energy up to $2\rho_0$. How well can Eq. (4) and Eq. (5) approximate the corresponding values at densities deviated from ρ_0 in various energy density functionals? Shown in Fig. 2 is $E_{\text{sym}}(\rho)$ vs $E_{\text{sym}}^*(\rho)$ with the 60 interactions at $\rho = 0.5\rho_0$, $\rho = 2\rho_0$, $\rho = 2.5\rho_0$ and $\rho = 3\rho_0$. A very strong linear correlation (the Pearson linear correlation coefficient r is larger than 0.98 for all the cases) is observed between $E_{\text{sym}}(\rho)$ and $E_{\text{sym}}^*(\rho)$ for the 60 interactions at all the four densities considered here. Similarly, Fig. 3 shows the correlation between $L(\rho)$ and $L^*(\rho)$ with the 60 interactions at $\rho = 0.5\rho_0$, $\rho = 2\rho_0$, $\rho = 2.5\rho_0$ and $\rho = 3\rho_0$, and again a strong linear correlation is observed between $L(\rho)$ and $L^*(\rho)$ (r is larger than 0.93 for the densities considered here).

The strong linear correlation between $E_{\text{sym}}(\rho)$ and $E_{\text{sym}}^*(\rho)$ as well as between $L(\rho)$ and $L^*(\rho)$ for the 60 interactions at different densities shown in Fig. 2 and Fig. 3 leads to the following relations

$$E_{\text{sym}}(\rho) \approx A(\rho) + B(\rho)E_{\text{sym}}^*(\rho), \quad (6)$$

$$L(\rho) \approx A_L(\rho) + B_L(\rho)L^*(\rho). \quad (7)$$

The values of the coefficients $A(\rho)$, $B(\rho)$, $A_L(\rho)$ and $B_L(\rho)$ generally depend the density. In particular, one can see from Fig. 2 and Fig. 3 that $A(\rho)$ ($A_L(\rho)$) is generally nonzero and $B(\rho)$ ($B_L(\rho)$) usually deviates from unit, reflecting the higher-order effects beyond the expansion in Eq. (4) and Eq. (5). We note $A(\rho) \approx 0$ ($A_L(\rho) \approx 0$) and $B(\rho) \approx 1$ ($B_L(\rho) \approx 1$) for $\rho \approx \rho_0$ as expected. Eq. (6) and Eq. (7) establish the systematics of $E_{\text{sym}}(\rho)$ and $L(\rho)$ in terms of three characteristic parameters $E_{\text{sym}}(\rho_0)$, L and K_{sym} . It should be noted that in principle Eq. (7) can also be deduced from Eq. (6) according to the definition. We would like to point out the Eq. (6) for the systematics of $E_{\text{sym}}(\rho)$ can be safely applied in the density region from $\rho_0/5$ to $3\rho_0$ where the Pearson linear correlation coefficient r is always larger than 0.96. Similarly,

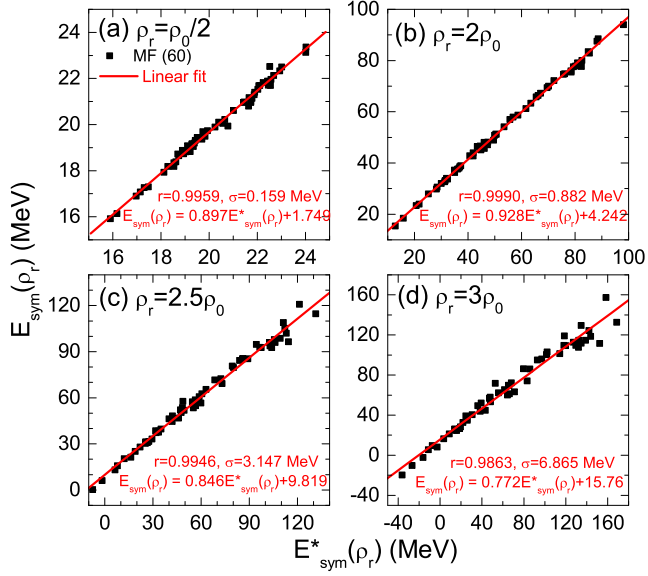


Figure 2. (Color online) $E_{\text{sym}}(\rho)$ vs $E^*_{\text{sym}}(\rho)$ in various energy density functionals with 60 interactions at $\rho = 0.5\rho_0$, $\rho = 2\rho_0$, $\rho = 2.5\rho_0$ and $\rho = 3\rho_0$.

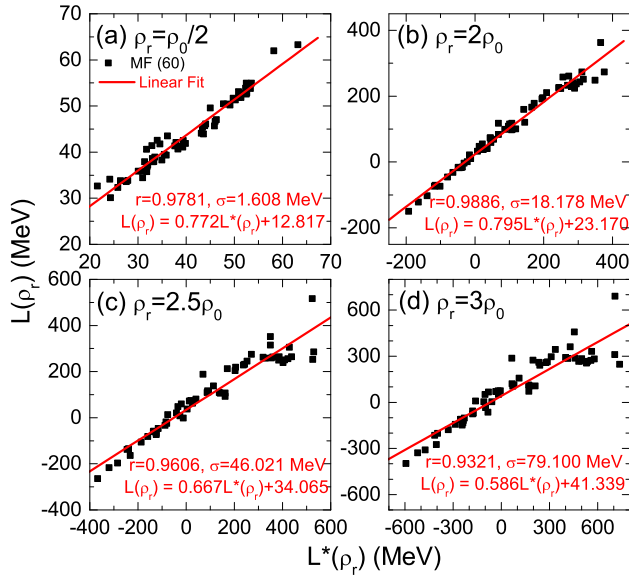


Figure 3. (Color online) Same as Fig. 2 but for $L(\rho)$ vs $L^*(\rho)$.

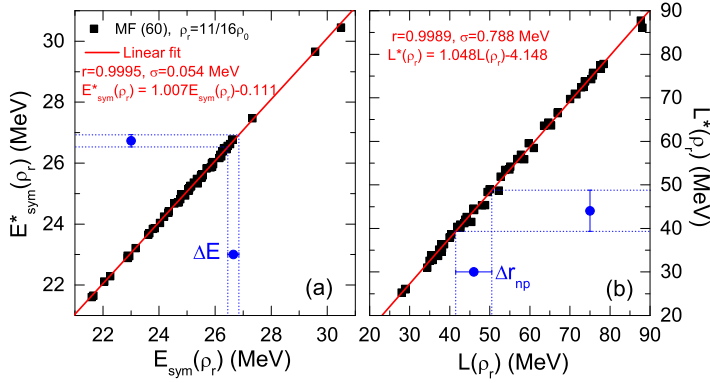


Figure 4. (Color online) $E_{\text{sym}}(\rho_c)$ vs $E_{\text{sym}}^*(\rho_c)$ (a) and $L(\rho_c)$ vs $L^*(\rho_c)$ (b) in various energy density functionals with the 60 interactions. The constraints $E_{\text{sym}}(\rho_c) = 26.65 \pm 0.20$ MeV (ΔE) and $L(\rho_c) = 46.0 \pm 4.5$ MeV (Δr_{np}) obtained in Ref. [37] are also included.

the Eq. (7) for the systematics of $L(\rho)$ can be safely applied in the density region from $\rho_0/2$ to $2.5\rho_0$ where the Pearson linear correlation coefficient r is always larger than 0.96.

3 Supra-saturation density behaviors of the symmetry energy

The systematics of $E_{\text{sym}}(\rho)$ and $L(\rho)$ in Eq. (6) and Eq. (7) imply that the three characteristic parameters $E_{\text{sym}}(\rho_0)$, L and K_{sym} (and thus $E_{\text{sym}}(\rho)$ and $L(\rho)$) can be determined once three values of either $E_{\text{sym}}(\rho)$ or $L(\rho)$ are known. This means that one can extract information on the high density behaviors of the symmetry energy from the relatively well constrained (sub-)saturation density behaviors of the symmetry energy.

In recent years, several accurate constraints on the symmetry energy at subsaturation density have been obtained through analyzing nuclear structure properties of heavy nuclei. Indeed, a quite accurate constraint on the symmetry energy at the subsaturation cross density $\rho_c = 0.11 \text{ fm}^{-3}$, i.e., $E_{\text{sym}}(\rho_c) = 26.65 \pm 0.20$ MeV, has been recently obtained from analyzing the binding energy difference of heavy isotope pairs [37]. At the same time, an accurate constraint on the density slope at ρ_c , i.e., $L(\rho_c) = 46.0 \pm 4.5$ MeV has been obtained from analyzing the neutron skin data of Sn isotopes [37]. At density $\rho_c = 0.11 \text{ fm}^{-3}$, the systematics of $E_{\text{sym}}(\rho)$ and $L(\rho)$ in Eq. (6) and Eq. (7) lead to the following expressions

$$E_{\text{sym}}^*(\rho_c) \approx a(\rho_c) + b(\rho_c)E_{\text{sym}}(\rho_c), \quad (8)$$

$$L^*(\rho_c) \approx a_L(\rho_c) + b_L(\rho_c)L(\rho_c). \quad (9)$$

The values of coefficients $a(\rho_c)$ and $b(\rho_c)$ ($a_L(\rho_c)$ and $b_L(\rho_c)$) can be obtained from linear fitting to the correlation between $E_{\text{sym}}^*(\rho_c)$ and $E_{\text{sym}}(\rho_c)$ ($L^*(\rho_c)$ and $L(\rho_c)$). Shown in Fig. 4 are $E_{\text{sym}}(\rho_c)$ vs $E_{\text{sym}}^*(\rho_c)$ and $L(\rho_c)$ vs $L^*(\rho_c)$ with the 60 interactions, and one can observe a very strong linear correlation (the Pearson linear correlation coefficient r is about 0.999 for both cases) between $E_{\text{sym}}^*(\rho_c)$ and $E_{\text{sym}}(\rho_c)$ as well as between $L^*(\rho_c)$ and $L(\rho_c)$, and these linear correlations lead to $a(\rho_c) = -0.111 \pm 0.111$ MeV, $b(\rho_c) = 1.007 \pm 0.004$, $a_L(\rho_c) = -4.148 \pm 0.358$ MeV and $b_L(\rho_c) = 1.048 \pm 0.006$.

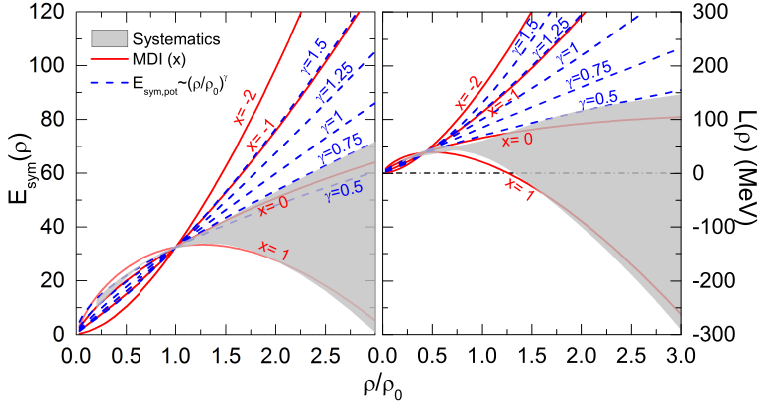


Figure 5. (Color online) Density dependence of $E_{\text{sym}}(\rho)$ and $L(\rho)$ according to the systematics in Eq. (6) and Eq. (7) with $E_{\text{sym}}(\rho_0) = 32.5 \pm 0.5$ MeV, $L = 46.7 \pm 12.8$ MeV and $K_{\text{sym}} = -166.9 \pm 168.3$ MeV. The results from the MDI interaction and the phenomenological parameterizations of $E_{\text{sym,pot}}(\rho) \sim (\rho/\rho_0)^\gamma$ are also include for comparison.

Besides $E_{\text{sym}}(\rho_c)$ and $L(\rho_c)$, one needs another constraint condition to determine $E_{\text{sym}}(\rho_0)$, L and K_{sym} and thus the supra-saturation density behaviors of the symmetry energy. In the present work, we further use the constraint of $E_{\text{sym}}(\rho_0) = 32.5 \pm 0.5$ MeV obtained recently from a new and more accurate finite-range droplet model analysis of the nuclear mass of the 2003 Atomic Mass Evaluation [38]. Therefore, from $E_{\text{sym}}(\rho_c) = 26.65 \pm 0.20$ MeV, $L(\rho_c) = 46.0 \pm 4.5$ MeV and $E_{\text{sym}}(\rho_0) = 32.5 \pm 0.5$ MeV, one can obtain $L = 46.7 \pm 12.8$ MeV and $K_{\text{sym}} = -166.9 \pm 168.3$ MeV. It is interesting to see that the obtained $L = 46.7 \pm 12.8$ MeV is in very good agreement with other constraints extracted from terrestrial experiments, astrophysical observations, and theoretical calculations with controlled uncertainties [13–18]. The obtained $K_{\text{sym}} = -166.9 \pm 168.3$ MeV also agrees well with the result $K_{\text{sym}} = -100 \pm 165$ MeV [29] obtained from a correlation analysis within SHF approach.

Based on $E_{\text{sym}}(\rho_0) = 32.5 \pm 0.5$ MeV, $L = 46.7 \pm 12.8$ MeV and $K_{\text{sym}} = -166.9 \pm 168.3$ MeV, one then can obtain $E_{\text{sym}}(\rho)$ and $L(\rho)$ according to the systematics in Eq. (6) and Eq. (7), and the results are shown in Fig. 5. For comparison, we also include the results from the MDI interaction [39] with $x = 1, 0, -1$ and -2 and the phenomenological parameterizations of $E_{\text{sym,pot}}(\rho) \sim (\rho/\rho_0)^\gamma$ [26, 40] for the potential energy part of the symmetry energy with $\gamma = 0.5, 0.75, 1.0, 1.25$ and 1.5 . We would like to point out that the MDI interaction and the parameterizations of $E_{\text{sym,pot}}(\rho) \sim (\rho/\rho_0)^\gamma$ have been extensively applied in transport model simulations of heavy ion collisions. One can see from Fig. 5 that the present analysis based on the symmetry energy systematics with $E_{\text{sym}}(\rho_0) = 32.5 \pm 0.5$ MeV, $L = 46.7 \pm 12.8$ MeV and $K_{\text{sym}} = -166.9 \pm 168.3$ MeV favors a softer symmetry energy and suggests that the symmetry energy cannot be stiffer than a linear density dependence.

In particular, at the supra-saturation density of $2\rho_0$, we find $E_{\text{sym}}(2\rho_0) = 40.2 \pm 12.8$ MeV and $L(2\rho_0) = 8.9 \pm 108.7$ MeV. We note that these values are in nice agreement with the variational many-body theory calculation with WFF1 interaction [41] which can give a good description on the recent observation of heavy neutron stars with radius of $9.1^{+1.3}_{-1.5}$ km [42].

4 Quark matter symmetry energy

At extremely high baryon density, the main degree of freedom could be the deconfined quark matter rather than the confined baryon matter, and there the quark matter symmetry energy should be involved for the properties of isospin asymmetric quark matter (isospin symmetry is still satisfied in quark matter). The isospin asymmetric quark matter could be produced in ultra-relativistic heavy ion collisions induced by neutron-rich nuclei and it could also exist in compact stars such as neutron stars or quark stars. Although significant progress has been made in understanding the density dependence of the nuclear matter symmetry energy, there has little information on the density dependence of the quark matter symmetry energy. Theoretically, it is difficult to calculate the quark matter symmetry energy since the *ab initio* Lattice QCD simulations does not work at finite baryon density while perturbative QCD only works at extremely high baryon density.

Similarly as in the case of nuclear matter, the EOS of quark matter consisting of u , d , and s quarks, defined by its binding energy per baryon number, can be expanded in isospin asymmetry δ_q as

$$E(n_B, \delta, n_s) = E_0(n_B, n_s) + E_{\text{sym}}(n_B, n_s)\delta_q^2 + \mathcal{O}(\delta_q^4), \quad (10)$$

where $E_0(n_B, n_s) = E(n_B, \delta_q = 0, n_s)$ is the binding energy per baryon number in three-flavor u - d - s quark matter with equal fraction of u and d quarks; the quark matter symmetry energy $E_{\text{sym}}(n_B, n_s)$ is expressed as

$$E_{\text{sym}}(n_B, n_s) = \frac{1}{2!} \left. \frac{\partial^2 E(n_B, \delta, n_s)}{\partial \delta_q^2} \right|_{\delta_q=0}. \quad (11)$$

The isospin asymmetry of quark matter is defined as

$$\delta_q = 3 \frac{n_d - n_u}{n_d + n_u}, \quad (12)$$

which equals to $-n_3/n_B$ with the isospin density $n_3 = n_u - n_d$ and $n_B = (n_u + n_d)/3$ for two-flavor u - d quark matter. We note that the above definition of δ_q for quark matter has been extensively used in the literature [43–47], and one has $\delta_q = 1$ (-1) for quark matter converted by pure neutron (proton) matter according to the nucleon constituent quark structure, consistent with the conventional definition for nuclear matter, namely, $\frac{\rho_n - \rho_p}{\rho_n + \rho_p} = -n_3/n_B$. In Eq. (10), the absence of odd-order terms in δ_q is due to the exchange symmetry between u and d quarks in quark matter when one neglects the Coulomb interaction among quarks. The higher-order coefficients in δ_q are shown to be very small in various model calculations [47].

It has been demonstrated recently [47] that the isovector properties of quark matter may play an important role in understanding the properties of strange quark matter and quark stars. If the recently discovered heavy pulsars PSR J1614-2230 [48] and PSR J0348+0432 [49] with mass around $2M_\odot$ were quark stars, they can put important constraint on the isovector properties of quark matter, especially the quark matter symmetry energy. Within the confined-isospin-density-dependent-mass (CIDDm) model [47], in particular, it has been shown that the two-flavor u - d quark matter symmetry energy should be at least about twice that of a free quark gas or normal quark matter within conventional NJL model in order to describe the PSR J1614-2230 and PSR J0348+0432 as quark stars.

5 Summary

We have explored the systematics of the density dependence of the symmetry energy in the ambit of microscopic calculations with various energy density functionals. Our results indicate that the symmetry energy magnitude $E_{\text{sym}}(\rho)$ and its density slope $L(\rho)$ from subsaturation density to supra-saturation density can be essentially determined by three parameters defined at saturation density ρ_0 , i.e., the magnitude $E_{\text{sym}}(\rho_0)$, the density slope L and the density curvature K_{sym} . This finding implies that three values of $E_{\text{sym}}(\rho)$ or $L(\rho)$ essentially determine $E_{\text{sym}}(\rho)$ and $L(\rho)$ in large density region. In particular, using $E_{\text{sym}}(\rho_c) = 26.65 \pm 0.2$ MeV and $L(\rho_c) = 46.0 \pm 4.5$ MeV at $\rho_c = 0.11 \text{ fm}^{-3}$ extracted from isotope binding energy difference and neutron skin of Sn isotopes together with $E_{\text{sym}}(\rho_0) = 32.5 \pm 0.5$ MeV obtained from finite-range droplet model analysis of nuclear binding energy, we obtain $L = 46.7 \pm 12.8$ MeV and $K_{\text{sym}} = -166.9 \pm 168.3$ MeV as well as $E_{\text{sym}}(2\rho_0) \approx 40.2 \pm 12.8$ MeV and $L(2\rho_0) \approx 8.9 \pm 108.7$ MeV. These results favor a soft to roughly linear density dependence of the symmetry energy.

We have also discussed the quark matter symmetry energy, which has been shown to play an important role in understanding the properties of strange quark matter and quark stars. In particular, it has been suggested that the two-flavor u - d quark matter symmetry energy should be at least about twice that of a free quark gas or normal quark matter within conventional NJL model in order to describe the recently discovered heavy pulsars PSR J1614-2230 and PSR J0348+0432 with mass around $2M_{\odot}$ as quark stars.

The author thanks Wei-Zhou Jiang, Che Ming Ko, Bao-An Li, De-Hua Wen, Hermann Wolter, and Jun Xu for useful discussions. This work was supported in part by the Major State Basic Research Development Program (973 Program) in China under Contracts No. 2015CB856904 and No. 2013CB834405, the National Natural Science Foundation of China under Grants No. 11275125 and No. 11135011, the “Shu Guang” project supported by Shanghai Municipal Education Commission and Shanghai Education Development Foundation, the Program for Professor of Special Appointment (Eastern Scholar) at Shanghai Institutions of Higher Learning, and the Science and Technology Commission of Shanghai Municipality (11DZ2260700).

References

- [1] B.A. Li, C.M. Ko, and W. Bauer, *Int. Jour. Mod. Phys. E* **7**, 147 (1998).
- [2] P. Danielewicz, R. Lacey, and W.G. Lynch, *Science* **298**, 1592 (2002).
- [3] J.M. Lattimer and M. Prakash, *Science* **304**, 536 (2004); *Phys. Rep.* **442**, 109 (2007).
- [4] W. Steiner, M. Prakash, J.M. Lattimer, and P.J. Ellis, *Phys. Rep.* **411**, 325 (2005).
- [5] V. Baran, M.Colonna, V. Greco, and M. Di Toro, *Phys. Rep.* **410**, 335 (2005).
- [6] L.W. Chen, C.M. Ko, B.A. Li, and G.C. Yong, *Front. Phys. China* **2**, 327 (2007).
- [7] B.A. Li, L.W. Chen, and C.M. Ko, *Phys. Rep.* **464**, 113 (2008).
- [8] C.J. Horowitz, S.J. Pollock, P.A. Souder and R. Michaels, *Phys. Rev. C* **63**, 025501 (2001).
- [9] T. Sil, M. Centelles, X. Viñas, and J. Piekarewicz, *Phys. Rev. C* **71**, 045502 (2005).
- [10] D.H. Wen, B.A. Li, and L.W. Chen, *Phys. Rev. Lett.* **103**, 211102 (2009).
- [11] H. Zheng, Z. Zhang, and L.W. Chen, *J. Cosmol. Astropart. Phys.* **08**, 011 (2014).
- [12] H. Zheng, K.J. Sun, and L.W. Chen, *Astrophys. J.*, 2015 (in press) [arXiv:1408.2926].
- [13] Topical Issue Nuclear Symmetry Energy edited by Bao-An Li, Angels Ramos, Giuseppe Verde, Isaac Vidana, *Eur. Phys. J. A* **50**, (2014).

- [14] B.M. Tsang *et al.*, Phys. Rev. C **86**, 015803 (2012).
- [15] J.M. Lattimer, Ann. Rev. Nucl. Part. Sci. **62**, 485 (2012).
- [16] L.W. Chen, Nuclear Structure in China 2012: Proceedings of the 14th National Conference on Nuclear Structure in China (NSC2012) (World Scientific, Singapore, 2012), pp. 43-54 [arXiv:1212.0284].
- [17] B.A. Li *et al.*, J. Phys.: Conf. Series **413**, 012021 (2013) [arXiv:1212.1178].
- [18] C.J. Horowitz *et al.*, J. Phys. G **41**, 093001 (2014).
- [19] Z. Xiao, B.A. Li, L.W. Chen, G.C. Yong, and M. Zhang, Phys. Rev. Lett. **102**, 062502 (2009); M. Zhang *et al.*, Phys. Rev. C **80**, 034616 (2009).
- [20] Z.Q. Feng and G.M. Jin, Phys. Lett. **B683**, 140 (2010).
- [21] J. Xu, C.M. Ko, Y. Oh, Phys. Rev. C **81**, 024910 (2010).
- [22] W.J. Xie, J. Sun, L. Zhu, and F.S. Zhang, Phys. Lett. **B718**, 1510 (2013).
- [23] J. Xu, L.W. Chen, C.M. Ko, B.A. Li, and Y.G. Ma, Phys. Rev. C **87**, 067601 (2013).
- [24] J. Hong and P. Danielewicz, Phys. Rev. C **90**, 024605 (2014).
- [25] T. Song and C.M. Ko, arXiv:1403.7363.
- [26] P. Russotto *et al.*, Phys. Lett. **B697**, 471 (2011).
- [27] M.D. Cozma, Y. Leifels, W. Trautmann, Q. Li, and P. Russotto, Phys. Rev. C **88**, 044912 (2013).
- [28] L.W. Chen *et al.*, Phys. Rev. C **80**, 014322 (2009).
- [29] L.W. Chen, Sci. China: Phys. Mech. Astro. **54** (Suppl. 1), s124 (2011).
- [30] D.H. Youngblood, H.L. Clark, and Y.-W. Lui, Phys. Rev. Lett. **82**, 691 (1999).
- [31] B.J. Cai and L.W. Chen, arXiv:1402.4242.
- [32] L.W. Chen, Sci. China Ser. G **52**, 1494 (2009).
- [33] J. Xu, L.W. Chen, B.A. Li, and H.R. Ma, Phys. Rev. C **79**, 035802 (2009).
- [34] J. Xu, L.W. Chen, B.A. Li, and H.R. Ma, Astrophys. J. **697**, 1549 (2009).
- [35] C. Ducoin, J. Margueron, and C. Providência, Europhys. Lett. **91**, 32001 (2010).
- [36] X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda, Phys. Rev. Lett. **106**, 252501 (2011).
- [37] Z. Zhang and L.W. Chen, Phys. Lett. **B726**, 234 (2013).
- [38] P. Moller, W.D. Myers, H. Sagawa, and S. Yoshida, Phys. Rev. Lett. **108**, 052501 (2012).
- [39] L.W. Chen, C.M. Ko, and B.A. Li, Phys. Rev. Lett. **94**, 032701 (2005).
- [40] M.B. Tsang *et al.*, Phys. Rev. Lett. **102**, 122701 (2009).
- [41] R.B. Wiringa, V. Fiks, and A. Fabrocini, Phys. Rev. C **38**, 1010 (1988).
- [42] S. Guillot, M. Servillat, N.A. Webb, and R.E. Rutledge, Astrophys. J. **772**, 7 (2013).
- [43] M. Di Toro, A. Drago, T. Gaitanos, V. Greco, and A. Lavagno, Nucl. Phys. **A775**, 102 (2006).
- [44] G. Pagliara and J. Schaffner-Bielich, Phys. Rev. D **81**, 094024 (2010).
- [45] M. Di Toro, V. Baran, M. Colonna, and V. Greco, J. Phys. G **37**, 083101(2010).
- [46] G.Y. Shao, M. Colonna, M. Di Toro, B. Liu, and F. Matera, Phys. Rev. D **85**, 114017 (2012).
- [47] P.C. Chu and L.W. Chen, Astrophys. J. **780**, 135 (2014).
- [48] P. Demorest, T. Pennucci, S. Ransom, M. Roberts, and J. Hessels, Nature **467**, 1081 (2010).
- [49] J. Antoniadis *et al.*, Science **340**, 6131 (2013).